Exam. Code : 103202
Subject Code : 1025

B.A./B.Sc. 2nd Semester

MATHEMATICS

Paper—I (Calculus and Differential Equations)

Time Allowed—2 Hours] [Maximum Marks—50

Note :— There are ***eight*** questions of equal marks. Candidates are required to attempt any ***four*** questions.

1. (a) Show that the asymptotes of the curve :

$$x^4 - 5x^2y^2 + 4y^4 + x^2 - y^2 + x + y + 1 = 0$$

Cut the curve in atmost eight points which lie on a rectangular hyperbola.

(b) Find the intervals in which the curve

$y = (\cos x + \sin x)e^x$ is concave upwards or downwards in $(0, 2\pi)$. Find also the points of inflexion.

2. (a) Find the position and nature of double points on the curve :

$$(2y + x + 1)^2 - 4(1 - x)^5 = 0$$

(b) Trace the curve $x^3 + y^3 = 3axy$, $a > 0$. 5+5=10

3. (a) Evaluate $\int \frac{\cosh x + \sinh x}{\cosh^2 x + \sinh^2 x} dx$

(b) Prove that

$$\frac{2}{\pi}\int_0^{\pi/2}\frac{1}{\sqrt{1-e^2\sin^2 x}}dx = 1+\frac{1^2}{2^2}e^2+\frac{1^2.3^2}{2^2.4^2}e^4+\frac{1^2.3^2.5^2}{2^2.4^2.6^2}e^6+\ldots\ldots$$

where $0 < e < 1$.

4. (a) Find the entire length of the curve $\left(\frac{x}{a}\right)^{\frac{2}{3}}+\left(\frac{y}{b}\right)^{\frac{2}{3}}=1$.

(b) Prove that the area of the curve $a^4y^2 = x^5(2a-x)$ to that of a circle of radius 'a' $(a > 0)$ is as 5 : 4.

5. (a) Solve $x^2-\frac{xy}{p}=f(y^2-xyp)$ where $p=\frac{dy}{dx}$.

(b) Solve $(xy^2 + 2x^2y^3)dx + (x^2y - x^3y^2)dy = 0$.

6. (a) Find the complete primitive and singular solution of $(a^2 - x^2)\left(\frac{dy}{dx}\right)^2+2xy\frac{dy}{dx}+(b^2-y^2)=0$.

(b) Show that the orthogonal trajectory of a system of concurrent straight lines is a system of concentric circles and conversely.

7. (a) Solve $(D^2 - 3D + 2)y = \cos(e^{-x})$ where $\frac{d}{dx}=D$ by method of variation of parameters.

(b) Solve $\frac{d^4y}{dx^4}-y=x^2\sin x$.

8. (a) Solve in series the differential equation :

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\left(x^2-\frac{1}{16}\right)y=0$$

(b) Solve the differential equation :

$$(x^2D^2+3xD+1)y=(1-x)^{-2} \text{ where } D=\frac{d}{dx}.$$